Guaranteed Safe Spacecraft Docking with Control Barrier Functions

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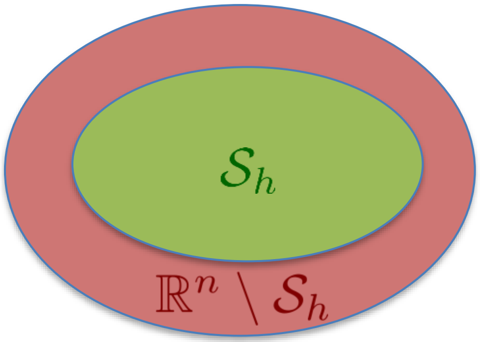
Objective

- Develop controllers for spacecraft docking that are:
  - Autonomous w.r.t. crew/ground control
  - Computationally lightweight
  - Provably safe
  - Input constrained
  - Robust to bounded disturbances

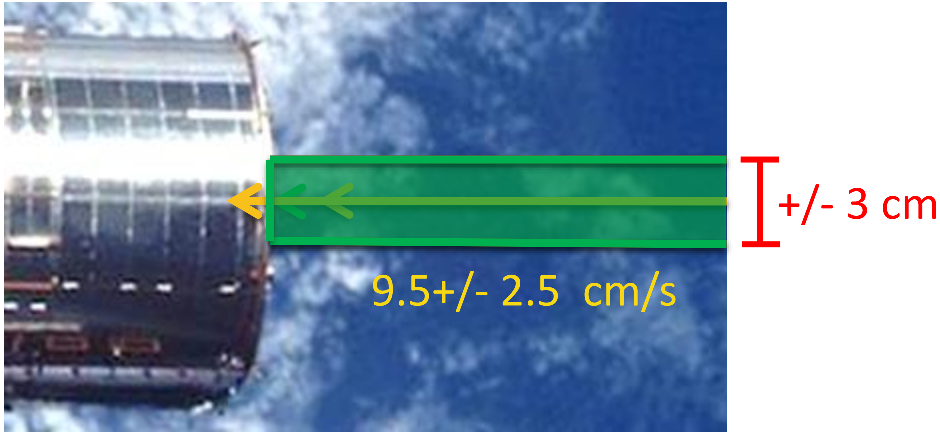
What is Safety?

- A system is called "safe" at time t if its state $x(t) \in \mathbb{R}^n$ belongs to a designated safe set $\mathcal{S}_h(t) \subset \mathbb{R}^n$ (potentially time-varying)
- In this paper, "safety" = "meets requirements"



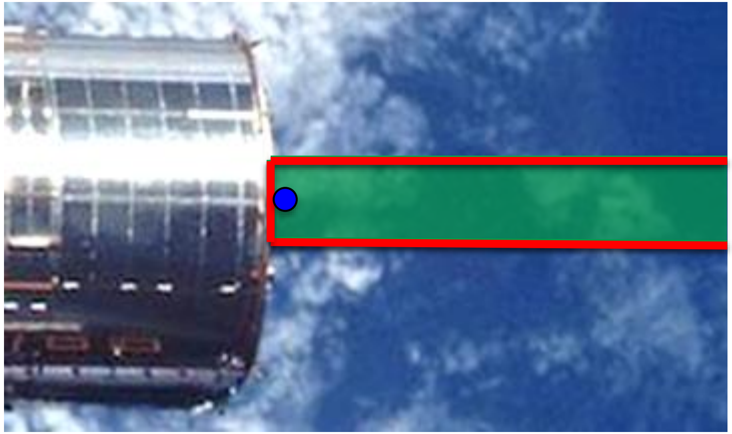
Safe Spacecraft Docking

- “Safety” = “meets requirements”
- Spacecraft docking has required tolerances
  - Narrow docking mechanism (cross-track, radial relative position)
  - Docking must occur within specified velocity tolerances (in-track velocity)
- Describe tolerances by a set $S_h \subset \mathbb{R}^n$



Safe Spacecraft Docking

- Spacecraft docking is a “tight tolerance” problem
  1. Safe set is small (in the context of the problem)
  2. Docking target lies close to the boundary of the safe set



Outline and Contributions

1. Achieving provable safety in the presence of input constraints and disturbances (see [6])
2. Extension of safety to allow for tight tolerance objectives
3. Application to spacecraft docking

[6] J. Breeden and D. Panagou, “Robust control barrier functions under high relative degree and input constraints for satellite trajectories,” Automatica, 2022, under review. [Online]. Available: https://arxiv.org/abs/2107.04094

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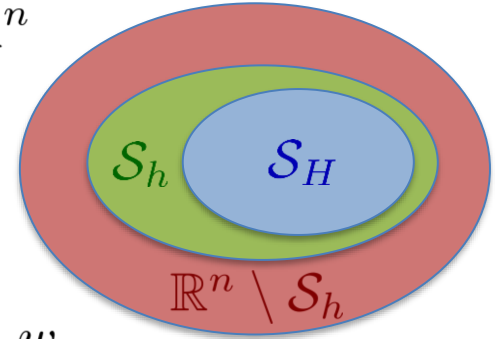
Primary Tool

- Control Barrier Functions (CBFs)
  - A CBF $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a tool for provably ensuring that the system state always lies within a designated safe set $\mathcal{S}_h \subset \mathbb{R}^n$
- Our formulation
  - State $x \in \mathbb{R}^n$, control $u \in \mathcal{U} \subset \mathbb{R}^m$, time $t \in \mathcal{T} \subseteq \mathbb{R}$
  - Dynamics $\dot{x} = f(t,x) + g(t,x)(u + w_u) + w_x$ with bounded disturbances $\|w_u\| \leq w_{u,\max}, \|w_x\| \leq w_{x,\max}$
  - Safe set: $\mathcal{S}_h(t) = \{x \in \mathbb{R}^n \mid h(t,x) \leq 0\}$ for a given function $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ of relative-degree two
  - Design a CBF H such that $\mathcal{S}_H(t) = \{x \in \mathbb{R}^n \mid H(t,x) \leq 0\}$ is a subset of $\mathcal{S}_h(t)$ and then render $\mathcal{S}_H$ forward invariant



Background – Control Barrier Functions

Definition. A $\mathcal{C}^1$ function $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a Control Barrier Function (CBF) on a set $\mathcal{X}$ if there exists a locally Lipschitz continuous $\alpha_0 \in \mathcal{K}$ such that $\forall x \in \mathcal{X}(t), t \in \mathcal{T}$,

$$\max_{\substack{\|w_u\| \leq w_{u,\max} \\ \|w_x\| \leq w_{x,\max}}} \inf_{u \in \mathcal{U}} \dot{H}(t, x, u, w_u, w_x) \leq \alpha_0(-H(t, x)) .$$

$$\dot{H}(t, x, u, w_u, w_x) = \underbrace{\partial_t H(t, x) + \nabla H(t, x) f(t, x)}_{\text{known, uncontrolled}} + \underbrace{\nabla H(t, x) g(t, x) u}_{\text{known, controlled}} + \underbrace{\nabla H(t, x) g(t, x) w_u + \nabla H(t, x) w_x}_{\text{unknown, bounded}}$$

(where $\mathcal{K}$ is the set of class-$\mathcal{K}$ functions $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$)

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$$\inf_{u \in \mathcal{U}} \dot{H}(t,x,u,0,0) + W(t,x) \leq \alpha_0(-H(t,x)) .$$

- Define

$$W(t,x) \triangleq \|\nabla H(t,x) g(t,x)\| w_{u,\max} + \|\nabla H(t,x)\| w_{x,\max}$$

which implies

$$\dot{H}(t,x,u,w_u,w_x)$$
$$\in [\dot{H}(t,x,u,0,0) - W(t,x),\ \dot{H}(t,x,u,0,0) + W(t,x)]$$

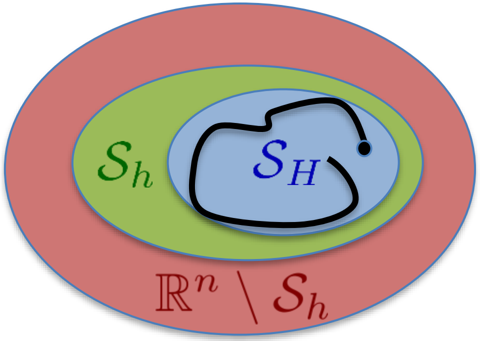
Background – Control Barrier Functions

Lemma ([6, Cor. 17]). Suppose $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ is a CBF on the set $\mathcal{S}_H$. Suppose there exists constants $\eta_1, \eta_2 > 0$ such that W satisfies $W(t,x) \in [\eta_1, \eta_2], \forall x \in \mathcal{S}_H(t), t \in \mathcal{T}$. Let $\alpha_w \in \mathcal{K}$ be locally Lipschitz continuous. Then any control law $u(t,x)$ that is piecewise continuous in t and locally Lipschitz continuous in x, and that satisfies: $\forall x \in \mathcal{S}_H(t), t \in \mathcal{T}$,

$$\dot{H}(t,x,u,0,0) \le \alpha_w(-H(t,x))W(t,x) - W(t,x) \quad (1)$$

will render the set $\mathcal{S}_H$ forward invariant.

- (1) is called the "CBF condition"
- $\dot{H}(t,x,u,0,0)$ is control-affine
- $\mathcal{S}_H$ is a viability domain



Background – Control Barrier Functions

- CBFs are composable using the CBF condition (1) repeatedly
- Implement controller as an LP or QP satisfying (1) for all i

$$u = \underset{\substack{u \in \mathcal{U} \\ \dot{H}_i \leq \alpha_w(-H_i)W - W,\, \forall i}}{\operatorname{argmin}} \quad u^{\mathrm{T}} J u + F u$$

- LP/QP with dimension m is computationally lightweight and constraints can be easily added/removed

CBFs for Input Constraints and Bounded Disturbances

- Inputs:
  - Safe set function: $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$
  - Control input constraints: $\mathcal{U}$
  - Disturbance bounds: $w_{u,\max}, w_{x,\max}$
  - Dynamics: f, g
- Assumptions – see [6]
- Outputs:
  - CBF: $H : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ such that $\mathcal{S}_H \subseteq \mathcal{S}_h$

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CBFs for Input Constraints and Bounded Disturbances

- Given $h : \mathcal{T} \times \mathbb{R}^n \to \mathbb{R}$ and under certain assumptions in [6, Thm. 9], the following is a CBF for <u>any $\alpha_0 \in \mathcal{K}$</u>

$$H(t,x) \triangleq \Phi^{-1}\left(\Phi(h(t,x)) - \frac{1}{2}\left|\dot{h}_w(t,x)\right|\dot{h}_w(t,x)\right) \tag{2}$$

$$\dot{h}_w(t,x) \triangleq \max_{\|w_x\| \leq w_{x,\max}} \dot{h}(t,x,w_x)$$

where $\Phi : \mathbb{R} \to \mathbb{R}$ is derived from the dynamics f and g, input constraints $\mathcal{U}$, and disturbance bounds $w_{u,\max}$ and $w_{x,\max}$

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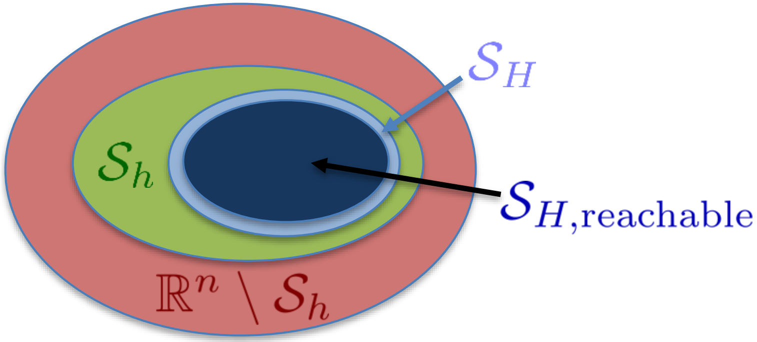
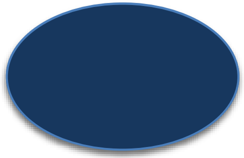
Problem with CBFs

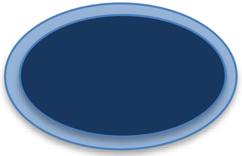
- Robustness to bounded disturbances introduces margins



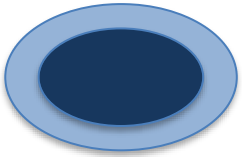
- The reachable safe set depends on the online disturbances w_u, w_x



Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = W(t,x)$$



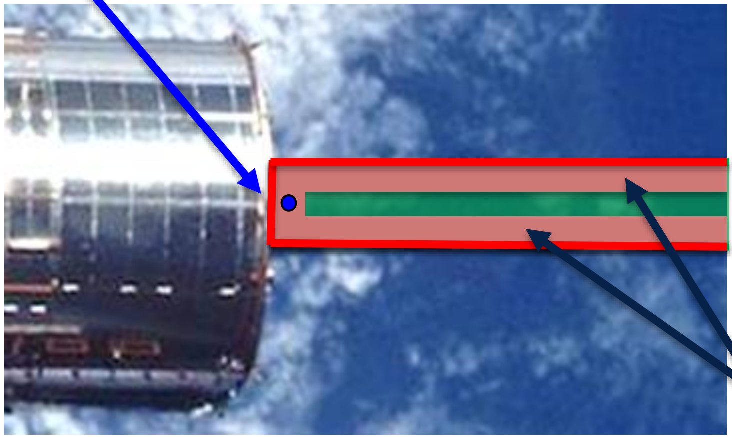
Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = 0$$



Reachable safe set if
$$\nabla H(t,x)g(t,x)w_u + \nabla H(t,x)w_x = -W(t,x)$$

Problem with CBFs

- The conservatism induced by (1) is problematic for tight tolerance objectives because
  1) The reachable safe set may become empty
  2) The target may not be inside the reachable safe set



Margins induced by robustness to worst-case $W(t, x)$

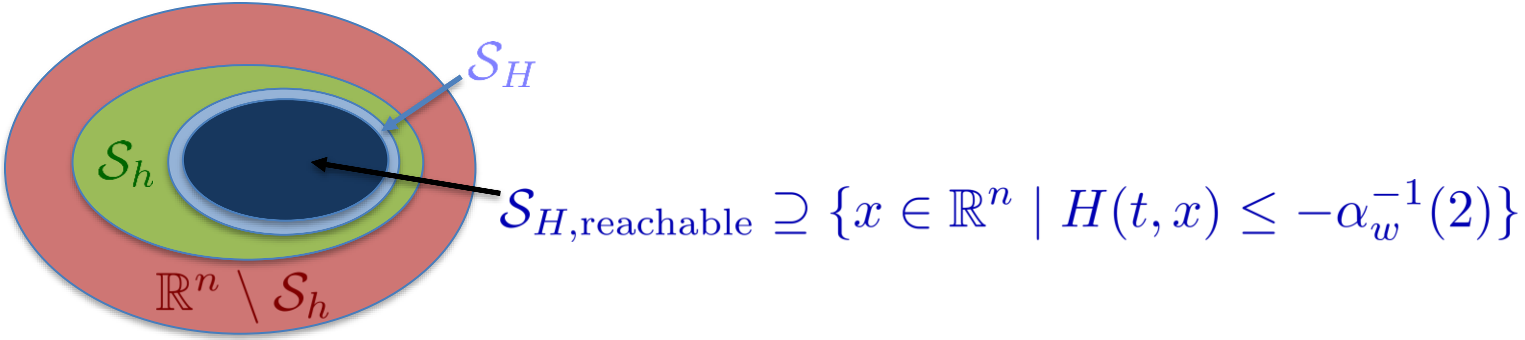
Tuning Robust CBF Margins

$$\dot{H}(t, x, u, 0, 0) \leq \alpha_w(-H(t, x))W(t, x) - W(t, x) \qquad (1)$$

- With H as in (2), we can choose any α_w

Lemma. If the control input $u(t, x)$ satisfies (1) with equality and $x(t_0) \in \mathcal{S}_H(t_0)$, then $\lim_{t\to\infty} H(t, x) \in [-\alpha_w^{-1}(2), 0]$.

- Choose α_w such that the "effective margin" $\alpha_w^{-1}(2)$ is sufficiently small



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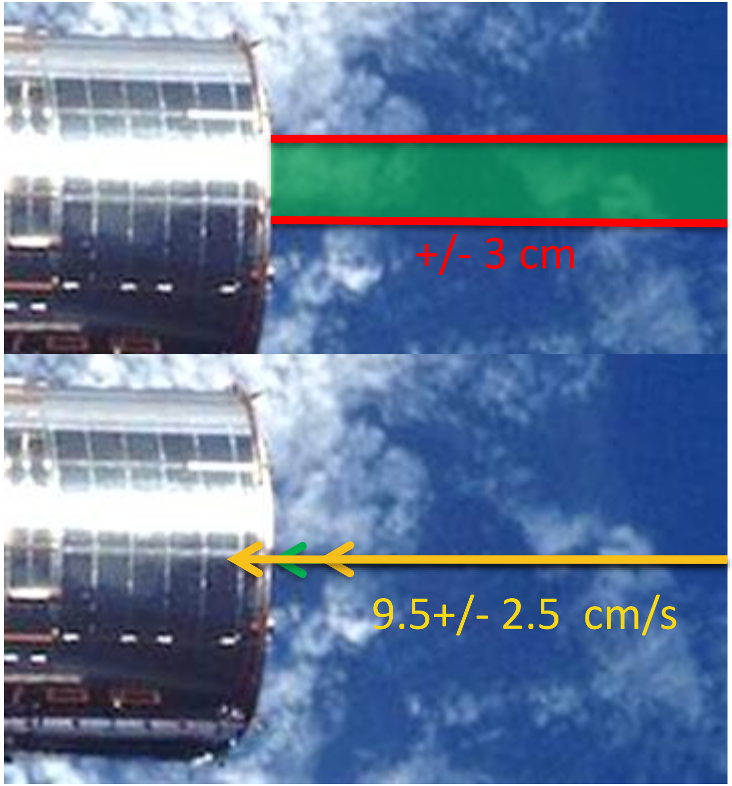
Docking Requirements

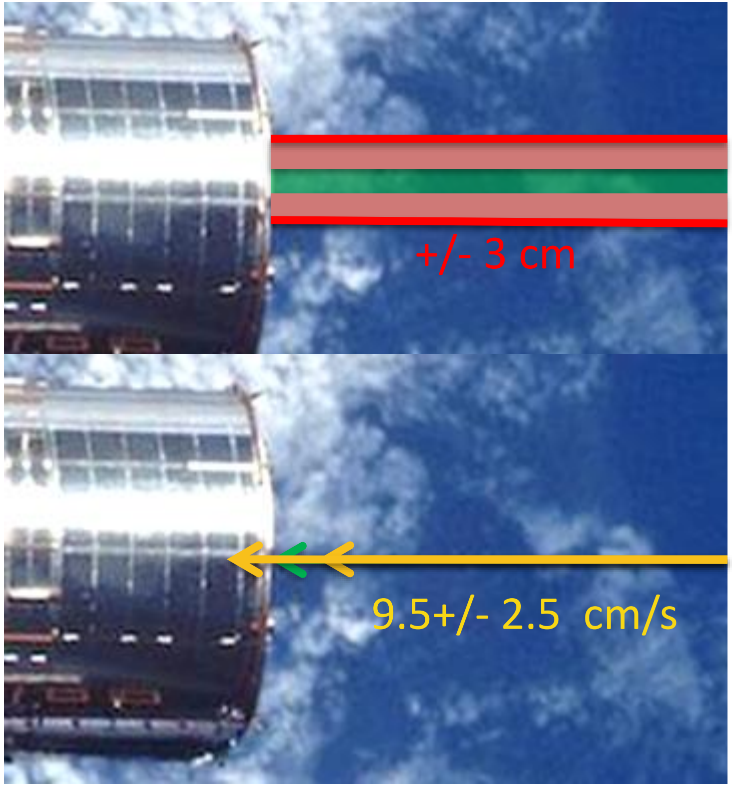
- Given $f, g, \mathcal{U}, w_{u,\max}, w_{x,\max}$



- Let h_l, h_r describe a docking cylinder
- Require $h_l(t, x(t)) \leq 0$ and $h_r(t, x(t)) \leq 0$ for all t

- Let h be the distance along the docking axis
- Require $h(t_f, x(t_f)) = 0$ and $\dot{h}(t_f, x(t_f)) \in [\gamma_1, \gamma_2]$ for some $t_f < \infty$

Docking Implementation



- Use prior lemma to ensure that $S_{H,\text{reachable}}$ is always nonempty

- Use prior lemma and Theorems 1-3 in paper (which relate H to h) to ensure docking axis requirements are satisfied in finite time

Simulations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & -2n & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} w_{x,1} \\ w_{x,2} \\ w_{u,1} \\ w_{u,2} \end{bmatrix}$$

$h(x) = -x_2 \quad \rightarrow H \text{ (Thm. 3)}$ (in-track distance)

$h_l(x) = x_1 - \Delta \quad \rightarrow H_l \text{ [6, Thm. 9]}$ (left radial constraint)

$h_r(x) = -x_1 - \Delta \quad \rightarrow H_r \text{ [6, Thm. 9]}$ (right radial constraint)

$H_v(x) = \|[\dot{x}_1, \dot{x}_2]\|_\infty - v_{max}$ (velocity constraint)

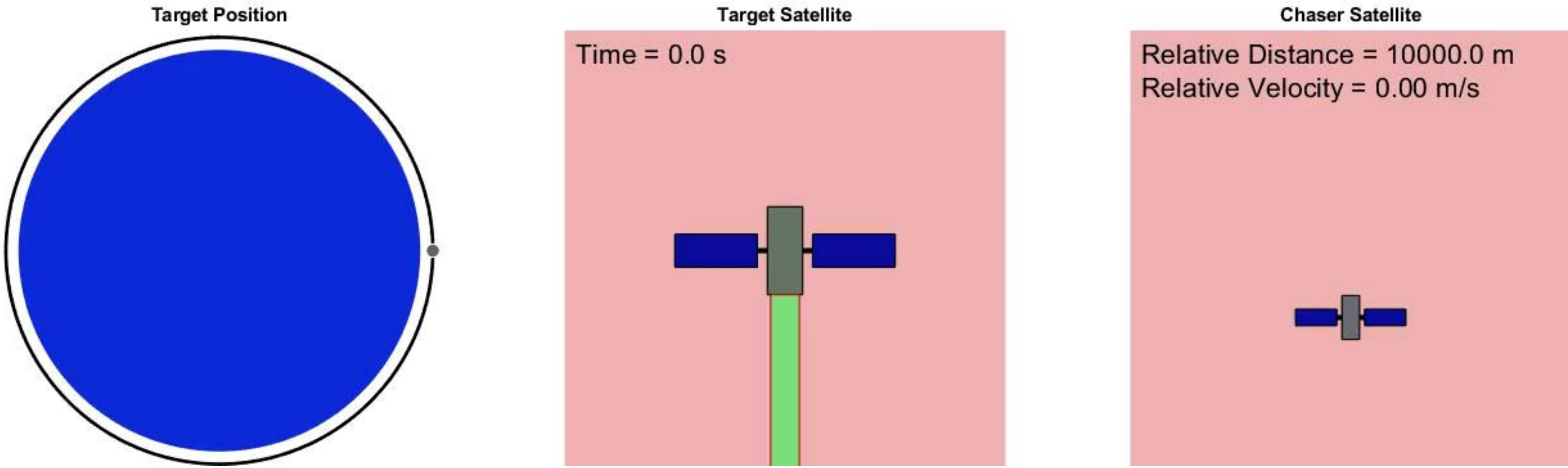
$$\Delta = 0.03 \text{ m}, \quad v_{max} = 10 \text{ m/s}, \quad \mathcal{U} = \{u \in \mathbb{R}^2 \mid \|u\|_\infty \leq 0.082 \text{ m/s}^2\}$$
$$w_{u,\max} = 0.002 \text{ m/s}^2, \quad w_{x,\max} = 0.001 \text{ m/s}$$

Simulations

$$u(t,x) = \begin{cases} \underset{\substack{u\in\mathcal{U}, \\ u \text{ satisfies (1) for } H, \\ u \text{ satisfies (1) for } H_r \\ u \text{ satisfies (1) for } H_v}}{\operatorname{argmin}} \|u - u_{nom}(t,x)\|^2 & H_l(t,x) > 0 \\ \underset{\substack{u\in\mathcal{U}, \\ u \text{ satisfies (1) for } H, \\ u \text{ satisfies (1) for } H_r, \\ u \text{ satisfies (1) for } H_l \\ u \text{ satisfies (1) for } H_v}}{\operatorname{argmin}} \|u - u_{nom}(t,x)\|^2 & H_l(t,x) \leq 0 \end{cases}$$

- u_{nom} is an attractive control law (drives x to the origin)
- h_l does not become active until the spacecraft first enters the safe set

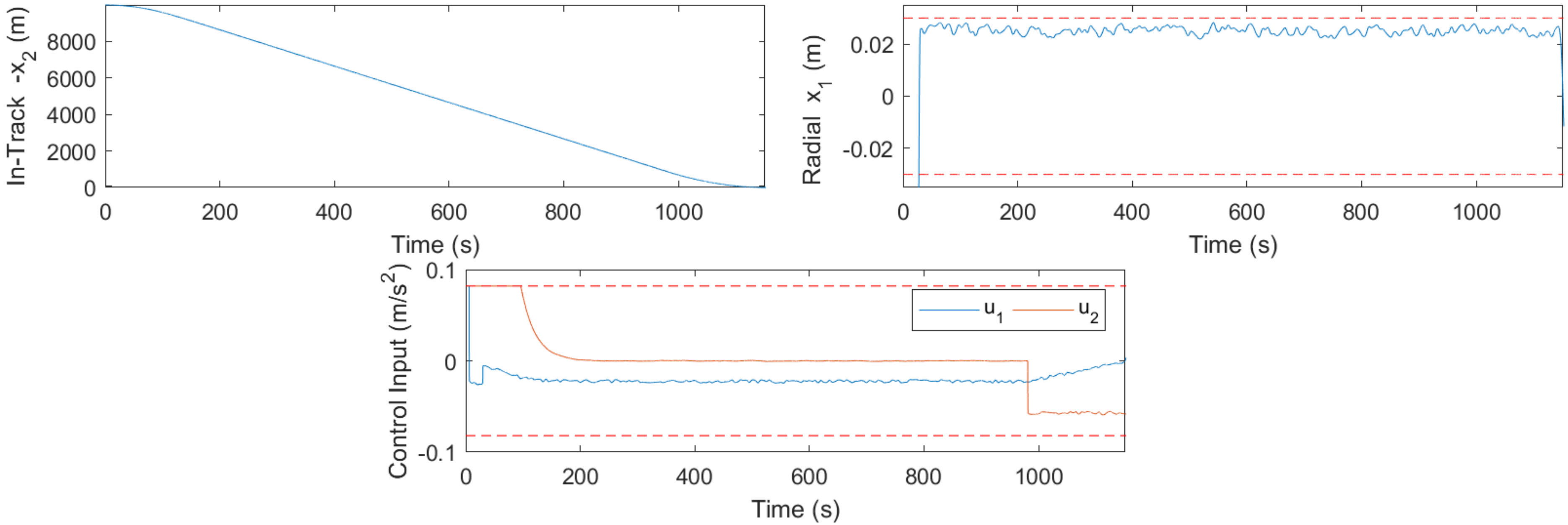
Simulation Results



(not to scale)

https://youtu.be/RoByiSD__jo

Simulation Results



- $\gamma_1 = 0.07\ \text{m/s},\ \gamma_2 = 0.12\ \text{m/s}$
- Docking velocity of $\dot{h}(t_f, x(t_f)) = 0.11\ \text{m/s}$

Conclusions

- CBFs are an effective methodology to represent spacecraft docking requirements
- The presented work allows tuning of CBF robustness margins while guaranteeing safety
- Future work:
  - Add additional constraints and realistic considerations:
    - Fuel efficiency
    - Obstacles
    - Fixed frequency controller
    - Measurement limitations

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Backup – Defining Docking

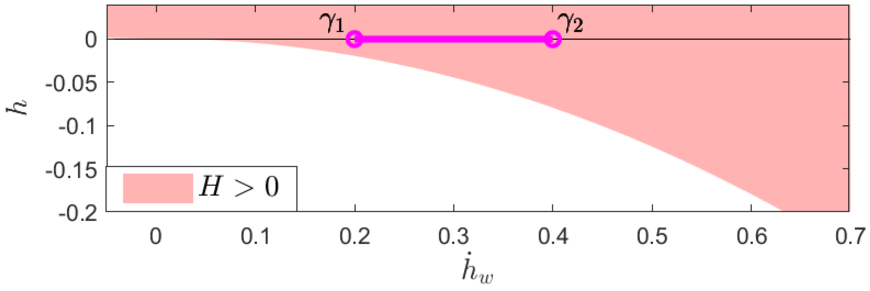
- Subproblems:
  1. Ensure $\dot{h}(t, x, w_x) \leq \gamma_2$ when $h(t, x) = 0$ (safety – Theorems 1-2)
     - Construct H from h using a form similar to (2)
  2. Ensure $h(t, x) = 0$ occurs in finite time (convergence – Theorem 3)
     - Satisfy (1) with equality and choose proper α_w
  3. Ensure $\dot{h}(t, x, w_x) \geq \gamma_1$ when $h(t, x) = 0$ (minimum energy – Corollary 1)
     - Define set of initial conditions where both velocity bounds are guaranteed

- Principal problem is relating the values of H to the values of h in order to use prior lemma

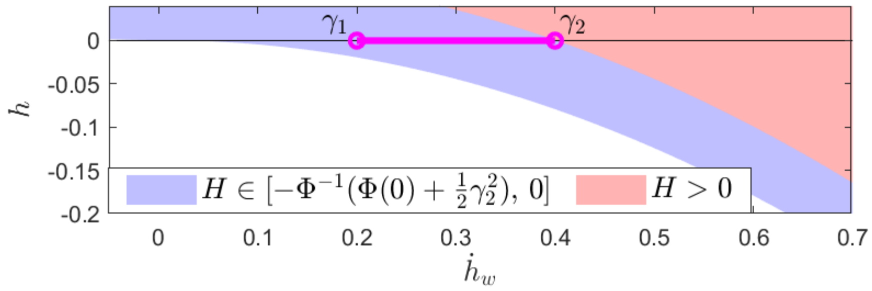
Backup – Solution to Subproblem 1

- Use the CBF (Theorems 1-2 in paper)

$$H(t,x) \triangleq \underbrace{\Phi^{-1}\left(\Phi(h(t,x)) - \frac{1}{2}\left|\dot{h}_w(t,x)\right|\dot{h}_w(t,x) +\right.}_{\text{from prior work}} \underbrace{\left.\frac{1}{2}\gamma_2^2\right)}_{\text{allowance for } \gamma_2}$$



Docking states are inaccessible because the prior work ensures $h \leq 0$



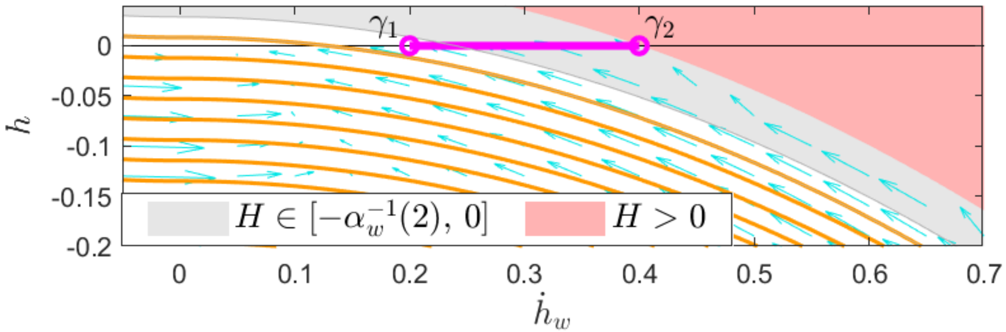
$\mathcal{S}_H$ now includes the blue region, which includes the docking states (magenta line)

Backup – Solution to Subproblem 2

Theorem. Suppose $\alpha_w^{-1}(2) = -\Phi^{-1}\left(\frac{1}{2}\gamma_2^2 + \Phi(0) - \frac{1}{2}(2l_h w_{x,\max} + \gamma_1)^2\right) > 0$. If the control input satisfies (1) with equality and $x(t_0) \in \mathcal{S}(t_0)$, then there exists finite $t_f > t_0$ such that $h(t_f, x(t_f)) = 0$.

- This relates the values of H to the values of h

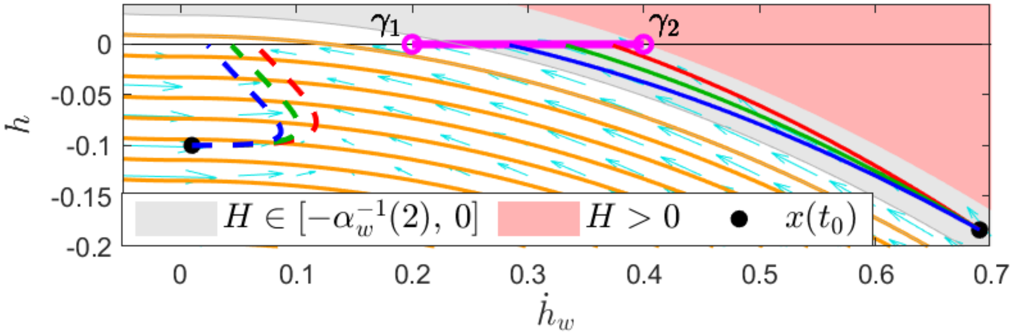
All trajectories satisfying (1) with equality reach the black line



Backup – Solution to Subproblem 3

Corollary. If additionally $H(t_0, x(t_0)) \geq -\alpha_w^{-1}(2)$, then $\dot{h}(t_f, x(t_f)) \geq \gamma_1$, i.e. docking is achieved.

All trajectories reach the black line but only trajectories inside the gray set are guaranteed to reach the magenta line



The colors correspond to different disturbances